FRQI With Multiple Color Palettes

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***Abstract*—One common criticism of the Flexible Representation of Quantum Images (FRQI) is the limited color palette. A new extension to FRQI is proposed that addresses the issue. This extension uses another clever technique in quantum computing known as superdense coding, which allows the transmission of classical bits using a lower number of qubits. A superdense coding scheme is used as a ”palette selector” where an FRQI state can choose which color palette should be used to decode its pixel information. FRQI images are separated into chunks where each chunk can have a different color palette. This extension only adds a few qubits to the entire image and allows for much more flexibility in FRQI.**

I. Introduction

Representing images in the quantum realm is a new challenge that is actively being researched. There are many different representation methods that have been developed such as the Flexible Representation of Quantum Images (FRQI), Novel Enhanced Quantum Representation (NEQR), Generalized Quantum Image Representation (GQIR), Multi-channel Representation for Quantum Images (MCQI), Quantum Boolean Image Processing (QBIP), and Entangled Shape Representation. Each of these has its own benefits and downsides. FRQI is a simple and easy to understand method that uses clever techniques to take advantage of quantum mechanics and encode images in a quantum state.

II. Background

A. Quantum Image Representation

The Flexible Representation of Quantum Images (FRQI) approach provides a way to map classical images into the quantum realm including color and pixel location. The idea is to encode an image in the state

$$|I(\theta)\rangle = \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} (\cos\theta_i |0\rangle + \sin\theta_i |1\rangle) \otimes |i\rangle \,,$$

$$\theta_i \in [0, \frac{\pi}{2}, \pi, i = 0, 1, \cdots, 2^{2n} - 1]$$

The advantage of this representation is its simplicity and the inclusion of color. Color is encoded in the first part of the state for each pixel, $\cos(\theta_i)|0\rangle + \sin(\theta_i)|1\rangle$, where θ_i represents some color. In theory there can be very many colors as θ can be anything, but in practice it is difficult to distinguish and maintain θ in a quantum state. Some implementations use more angles, but for simplicity $0, \frac{\pi}{2}, \pi$ will be used to encode only three colors in this paper.

Fig. 1. Grayscale color palette used with FRQI.

The second part of the FRQI state, $|i\rangle$, encodes the location information on each pixel. For example, in a 2x2 image $|i\rangle$ will take on the values $|0\rangle, |1\rangle, |2\rangle, |3\rangle$, where each qubit is represented in the following manner using the Zeeman basis:

$$|i\rangle \to |xy\rangle$$

$$|0\rangle \to |00\rangle$$

$$|1\rangle \to |01\rangle$$

$$|2\rangle \to |10\rangle$$

$$|3\rangle \to |11\rangle$$

$$\cdots$$

Constructing a FRQI state is simple and can be done with commonly available gates including Hadamard, controlled-rotation, not, and controlled-not gates. Note that the controlled rotation gates that will be used are of 2θ.

$$R_y(\theta) = \begin{pmatrix} \cos(\frac{\theta}{2}) & -\sin(\frac{\theta}{2}) \\ \sin(\frac{\theta}{2}) & \cos(\frac{\theta}{2}) \end{pmatrix}$$

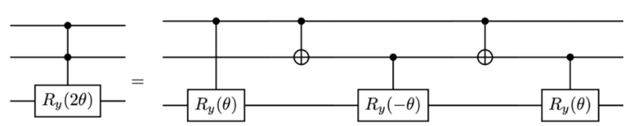

Fig. 2. R_y rotation gate and identity for a controlled rotation gate.

The general implementation of the quantum circuit used to encode a 2x2 image in FRQI form is:

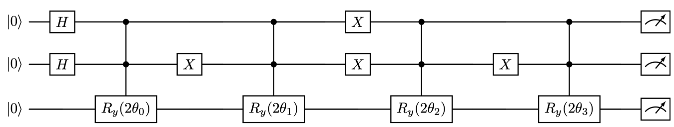

Fig. 3. Quantum circuit for a 2x2 FRQI state.

Each rotation gate corresponds to the color of each pixel. The X gates are used to encode pixel position. Take the 2x2 image shown below for example, as well as the quantum circuit that encodes that image. Note that the color palette shown in Figure 1 will be used.

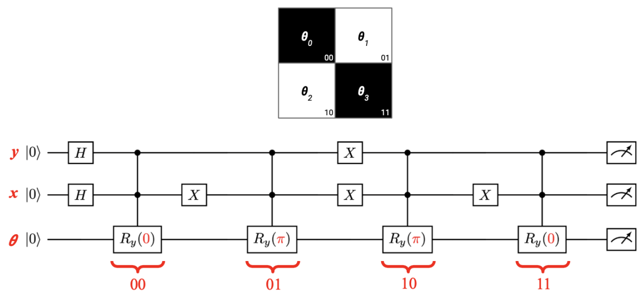

Fig. 4. FRQI Circuit to encode the image. Red text is used for clarification.

Walking through the circuit step by step can help show how the state evolves and how pixel information is encoded in the state. Take the example in Figure 4:

1. Place the position qubits in a state of superposition using Hadamard gates.

$$(I \otimes H^{\otimes 2}) \left|0^{\otimes 3}\right\rangle = \frac{1}{2} |0\rangle \otimes \sum_{i=0}^{3} |i\rangle = |H\rangle$$

2. Apply the controlled rotation gate,

$$R_i = (I \otimes \sum_{j=0, j\neq i}^{3} |i\rangle \langle i|) + R_y(2\theta_i) \otimes |i\rangle \langle i|$$

for each pixel.

$$R_0 |H\rangle = \frac{1}{2}\Big(|0\rangle \otimes \sum_{i=0,i\neq 0}^{3} |i\rangle \langle i| + (\cos(0)|0\rangle + \sin(0)|1\rangle) \otimes |0\rangle \Big)$$

$$R_1(R_0 |H\rangle) = \frac{1}{2}\Big(|0\rangle \otimes \sum_{i=0,i\neq 0,1}^{3} |i\rangle \langle i|$$

$$+ (\cos(0)|0\rangle + \sin(0)|1\rangle) \otimes |0\rangle$$

$$+ (\cos(\pi)|0\rangle + \sin(\pi)|1\rangle) \otimes |1\rangle \Big)$$

$$R_2(R_1(R_0 |H\rangle)) = \frac{1}{2}\Big(|0\rangle \otimes \sum_{i=0,i\neq 0,1,2}^{3} |i\rangle \langle i|$$

$$+ (\cos(0)|0\rangle + \sin(0)|1\rangle) \otimes |0\rangle$$

$$+ (\cos(\pi)|0\rangle + \sin(\pi)|1\rangle) \otimes |1\rangle$$

$$+ (\cos(\pi)|0\rangle + \sin(\pi)|1\rangle) \otimes |2\rangle \Big)$$

$$R_3(R_2(R_1(R_0 |H\rangle))) = \frac{1}{2}\Big(|0\rangle \otimes \sum_{i=0,i\neq 0,1,2,3}^{3} |i\rangle \langle i|$$

$$+ (\cos(0)|0\rangle + \sin(0)|1\rangle) \otimes |0\rangle$$

$$+ (\cos(\pi)|0\rangle + \sin(\pi)|1\rangle) \otimes |1\rangle$$

$$+ (\cos(\pi)|0\rangle + \sin(\pi)|1\rangle) \otimes |2\rangle$$

$$+ (\cos(0)|0\rangle + \sin(0)|1\rangle) \otimes |3\rangle \Big)$$

Now the image is encoded in the FRQI state. Notice that the summation can be discarded. After translating from the Zeeman basis the resulting final state is:

$$\frac{1}{2}\Big((\cos(0)|0\rangle + \sin(0)|1\rangle) \otimes |00\rangle$$

$$+ (\cos(\pi)|0\rangle + \sin\pi\,|1\rangle) \otimes |01\rangle$$

$$+ (\cos(\pi)|0\rangle + \sin\pi\,|1\rangle) \otimes |10\rangle$$

$$+ (\cos(0)|0\rangle + \sin(0)|1\rangle) \otimes |11\rangle \Big)$$

Recovering the pixel information from this state is simple. Every time the state is measured, the state will collapse into one of the pixels of the 2x2 image. Since every pixel has the same probability of being measured, repeated measurements will eventually reveal all four pixels. 1024 measurements seems to be enough to recover a 2x2 image.

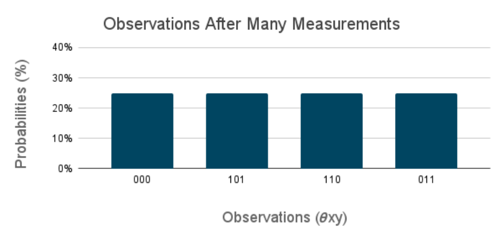

Fig. 5. Expected probabilities of observing pixel information from a 2x2 FRQI state.

Another benefit of FRQI states is the ability to compute rudimentary image processing through the application of unitary gates. The first example is in the following form:

$$G_1 = U \otimes I^{\otimes n}$$

This operator can be used to apply the gate U to all pixels. If we set $U = R_y(2\theta)$, the following results:

$$G_1 |I\rangle = \frac{1}{2^n} \sum_{i=0}^{2^{2n}-1} (\cos(\theta_i + \theta)|0\rangle + \sin(\theta_i + \theta)|1\rangle) \otimes |i\rangle$$

$|I\rangle$ is a FRQI state. Essentially this operator shifts the color of every pixel by θ. A more general operator is:

$$G_2 = U \otimes C + I \otimes \bar{C}$$

Where C denotes which pixels the gate U should be applied to and $\bar{C}$ is the complement of C. For example, setting $C = |0\rangle\langle 0| + |1\rangle\langle 1|$ and $\bar{C} = |2\rangle\langle 2| + |3\rangle\langle 3|$, the following state results:

$$\begin{aligned} G_2 |I\rangle = \frac{1}{2}\Big(& (\cos(\theta_0)U|0\rangle + \sin(\theta_0)U|1\rangle) \otimes |0\rangle \\ & + (\cos(\theta_1)U|0\rangle + \sin(\theta_1)U|1\rangle) \otimes |1\rangle \\ & + (\cos(\theta_2)|0\rangle + \sin(\theta_2)|1\rangle) \otimes |2\rangle \\ & + (\cos(\theta_3)|0\rangle + \sin(\theta_3)|1\rangle) \otimes |3\rangle \Big) \end{aligned}$$

So the gate U is only applied to the first and second pixels. Essentially, G_2 is a controlled gate.

The downsides of FRQI are worth mentioning. Some of the gates used might not be available in current quantum computers. Another problem in FRQI is that noise can lead to inconsistent image representations in current quantum computer technology due to the angles used for the color pixel selection. Images must be created by hand, pixel by pixel, by encoding the image directly into a quantum circuit, which might not be feasible for large images. The limited color palette has also deterred some from using FRQI.

B. Superdense Coding

Charles Bennet and Stephen Wiesner proposed a quantum coding scheme that can represent two classical bits only using one qubit. This scheme involves an oracle Apollo, a sender Alice, and a receiver Bob. Apollo sends a pair of entangled bits to Alice and Bob. The state is usually in Bell basis, for example $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$. Then Alice applies the quantum gates X^bZ^a to the least significant qubit, where a and b represent the classical bits. Alice can send two classical bits, which means that she can apply four different operations for X^bZ^a. The four possibilities, their corresponding operations, and the resulting states are summarized below:

ab	Operation	State
00	$X^0Z^0 = I$	$\|00\rangle + \|11\rangle$
01	$X^1Z^0 = X$	$\|01\rangle + \|10\rangle$
10	$X^0Z^1 = Z$	$\|00\rangle - \|11\rangle$
11	$X^1Z^1 = XZ$	$\|01\rangle - \|10\rangle$

The $\frac{1}{\sqrt{2}}$ normalization term for each state is not shown. Note that Bob must first convert back to the computational basis and not the Bell basis to recover a and b. This is simple, as it is the reverse of what Apollo did to create the Bell basis. This scheme is known as superdense coding. Below is a visual representation of the entire quantum circuit.

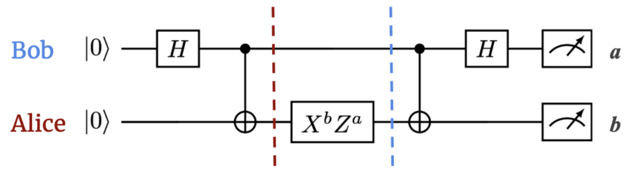

Fig. 6. Superdense coding circuit. Alice performs her operations after the red barrier. Bob measures after the blue barrier.

Superdense coding takes advantage of the fact that unitary gates can be used to convert from one state in the Bell basis to another, and Bob is able to decode based on the state he receives.

In the scheme proposed by Bennet and Wiesner, a pair of two entangled qubits are used. It turns out that superdense coding can be generalized to higher dimensions by allowing more than two entangled qubits to be used. Note that qubits must be maximally entangled for superdense coding, which means that measuring one qubit will determine the output of the other qubit.

Gorbachev et al., and Cereceda proposed a superdense coding scheme using three entangled qubits. This scheme uses two qubits to represent three classical bits. It is assumed that Apollo distributes three entangled qubits between Alice and Bob. Alice begins with a state in the GHZ basis: $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$. Then in a similar manner as in Bennet and Wisener's scheme, Alice applies certain gates to her two qubits, which determine which classical bits she wants to send. Alice can send up to 8 different states, which correspond to three classical bits, using two qubits. The states and operators Alice will perform are neatly summarized below:

abc	A	B	$AB\|GHZ\rangle$
000	I	I	$\|000\rangle + \|111\rangle$
001	I	σ_x	$\|001\rangle + \|110\rangle$
010	σ_z	I	$\|000\rangle - \|111\rangle$
011	σ_z	σ_x	$\|001\rangle - \|110\rangle$
100	σ_x	I	$\|010\rangle + \|101\rangle$
101	σ_x	σ_x	$\|011\rangle + \|100\rangle$
110	$-i\sigma_y$	I	$\|010\rangle - \|101\rangle$
111	$-i\sigma_y$	σ_x	$\|011\rangle - \|100\rangle$

The first column corresponds to the three classical bits Alice will send, the second and third columns are the Pauli gates which Alice will apply to her qubits, and the last column is the resulting GHZ state. Note that the normalization factor, $\frac{1}{\sqrt{2}}$, is omitted from the GHZ states for simplicity. It is possible further extend these superdense coding schemes with more qubits.

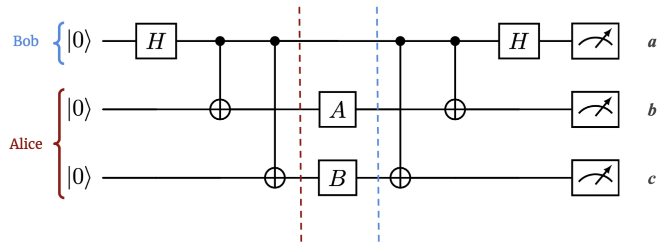

Fig. 7. Superdense coding scheme with three qubits.

Alexander Holevo proved that there is a limit to the amount of classical information that a quantum channel can carry. Suppose that the amount of classical information that we want to send through a channel is I. Then the inequality:

$$I \leq S(\rho)$$

must be true. $S(\rho)$ is the von Neumann entropy is defined as $-Tr(\rho log \rho)$. This is important for superdense coding as it sets the absolute limit on the amount of classical bits that can be sent through a quantum channel using schemes like superdense coding.

III. Motivation

Representing images using FRQI is very neat because only a few qubits are required to encode an image that would require more bits classically. Images can be as large as necessary as shown by the general definition of the FRQI state mentioned in the background section. One downside of FRQI mentioned before is the limited color palette that is available. Only a few colors can feasibly be represented due to current quantum computing limitations. The idea is to get around this color limitation using superdense coding. With a few more qubits, it is possible to create a "palette selector" where the image can also encode which color palette it will use.

IV. Method

Suppose that Alice wants to encode an image using FRQI. This image can be larger than the 2x2 examples that have been shown before. She is able to do so, although the image will be limited to the grayscale color palette that is often chosen for FRQI images.

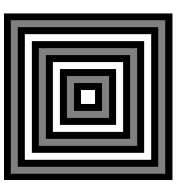

Fig. 8. Grayscale 24x24 FRQI image example.

Now suppose Alice wants to encode a 2x2 image and send it to Bob. Apollo distributes a pair of entangled qubits to Alice and Bob. Alice extends her FRQI image to include this new qubit. Now Alice is able to use her entangled bit to encode a color palette, using superdense coding. Alice can now choose from four different color palettes, each depending on which classical bits she chooses to encode.

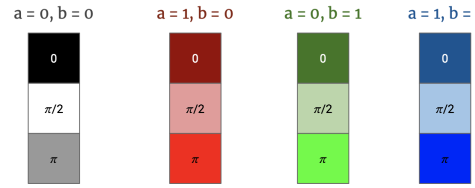

Fig. 9. More color palettes corresponding to superdense classical bits.

Once Alice has encoded her image, she sends her four qubits to Bob, who then is able to recover the color palette using the second portion of superdense coding and repeatedly measure the FRQI state to reconstruct the original image.

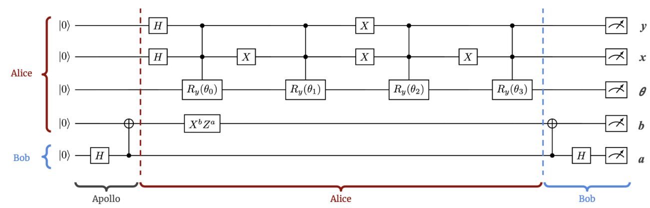

Fig. 10. FRQI circuit with superdense palette selector

This extended FRQI method allows for the representation of images with more colors. For larger than 2x2 images Alice must simply break up her image into 2^nx2^n chunks where each chunk has its own color palette. These chunks will then later be stitched together by Bob, using some protocol agreed upon beforehand. This means that Alice and Bob must share a pair of entangled qubits for each chunk.

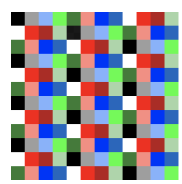

Fig. 11. 24x24 FRQI image with the palettes from Figure 9. Chunks are 2x2.

Using higher dimension superdense coding, such as using the GHZ states with three qubits, allows for even more colors, although with more qubits necessary for every chunk.

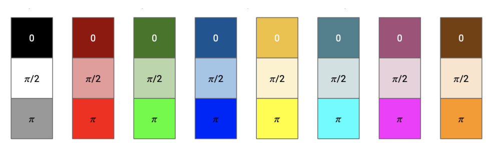

Fig. 12. Even more palettes when using GHZ superdense scheme.

V. Future Work

The flexibility of the FRQI state has been extended to include more color palettes using superdense coding schemes, but there are still more questions that could be answered. One problem with this new extension of FRQI is that even though images can use different color palettes, it is not possible to mix colors from different color palettes. This leads to the problem of choosing the optimal color palettes, depending on the situation and the type of images that will be represented. Since the color palettes are arbitrary, there is creative freedom, but this is still a problem that needs more research. Testing the limits for how many colors and color palettes are possible to implement in practice also needs more experimental research. Choosing the size of the chunks that Alice can use for each color palette is another interesting exploration that may be useful for larger images that have similar colors in adjacent pixels. This would be useful to reduce the number of entangled qubits necessary. FRQI's image processing abilities should also be further explored in the context of this new palette selector extension. Since Bob will have to measure the states sent by Alice many times to recover the image anyways, he can measure a, b to determine which color palette was sent and apply a unitary transformation to switch the palette if he wishes to do so.

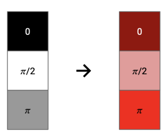

Fig. 13. Swapping from grayscale to red palette.

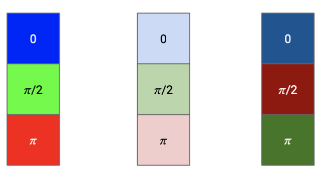

Fig. 14. Examples of alternate color palettes.

VI. Conclusion

FRQI's singular grayscale color palette was extended by using superdense coding schemes. A color palette selector was proposed where each palette corresponds to a state sent through a superdense coding scheme. This extends the ability of FRQI's image representation abilities while only adding a few more qubits for each chunk of pixels. Future work could further improve on this new extension and provide greater insights on what is possible using superdense coding as a color palette selector.

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